# Sample Constructed Response Question from
## *Preparing for the CSET – Mathematics Subtest ll*

Show that the area of the region shaded orange outside of the square is equal to the total area of the regions shaded blue inside of the square.

The Orange Area is defined as the area between a circle and the inscribed square.  The Blue Area is the overlapping area of four congruent semi-circles with each side of the square as a diameter of the semi-circle.
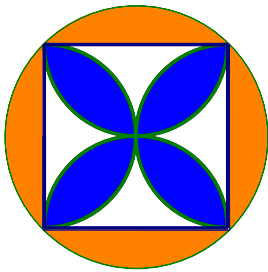


Solution begins on next page

# Solution

Show that the area of the region shaded orange outside of the square is equal to the total area of the regions shaded blue inside of the square.
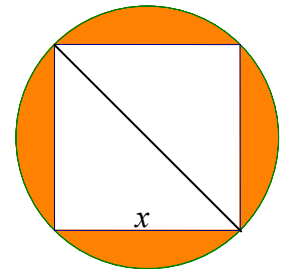
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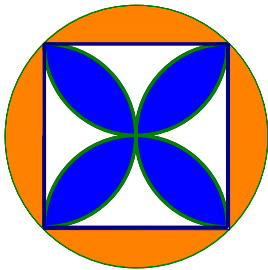


Start by finding the Orange Area.
To find the Orange Area, we must find the area of the circle and subtract from it the area of the square.



$$\bigcirc \; - \; \square \; = \; \bigcirc$$

If we define x as the length of the side of the square, then the area of the square would be $x^2$.

The diagonal of the square is also the diameter of the circle. Using the Pythagorean Theorem, we can find its length to be $x\sqrt{2}$. The radius is ½ of the diameter, or $\dfrac{x\sqrt{2}}{2}$.

The area of a circle is $\pi r^2$, or in this case $\pi\left(\dfrac{x\sqrt{2}}{2}\right)^2$, which simplifies to $\dfrac{\pi x^2}{2}$.

Since the Orange Area is the difference between the areas of the circle and square, it is $\dfrac{\pi x^2}{2} - x^2$ square units.

Now let's find the Blue Area.

Since the Blue area is formed by four overlapping semi-circles we should start by adding together their areas. (In these diagrams, dark shading represents areas that have been counted twice and light shading represents areas that have been counted once.)

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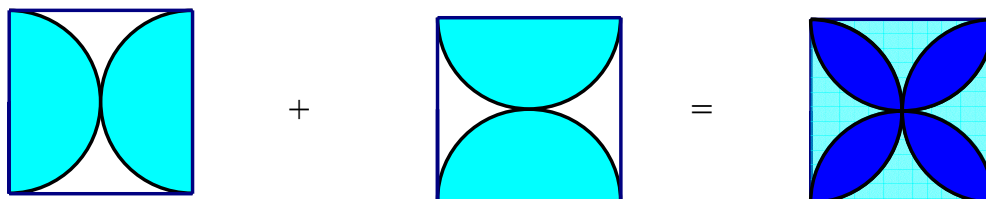
The problem with this is that we have double counted the area that we want (the dark blue flower) and single-counted the area that we didn't even want to count once (the light blue wedges).

If we take our above result and subtract the area of a square, we lose the single-counted wedges and reduce the double counted flower to the single-count that we want.

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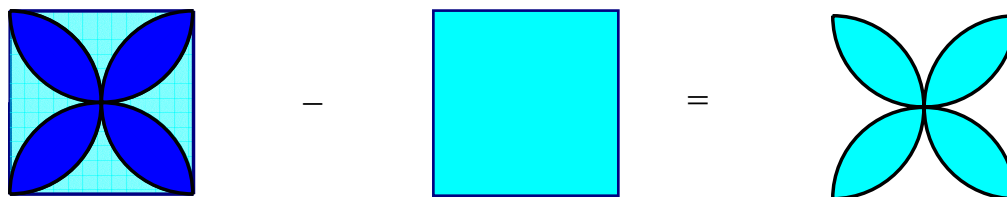
Numerically, the radius of each semi-circle is $\frac{x}{2}$. (The side of the square is the diameter of the semi-circle, so cut it in half to get the radius.) Therefore, the total area of the 4 semicircles would be $4\left(\frac{1}{2}\left[\pi\left(\frac{x}{2}\right)^2\right]\right)$ which simplifies to $\frac{\pi x^2}{2}$. (The ½ is in there because we are looking at semi-circles.)

When we then subtract the area of the square we get $\frac{\pi x^2}{2} - x^2$ square units, which is same as the Orange Area found above.

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