Sample Multiple Choice Question from
Preparing for the CSET – Mathematics Subtest l

Which of the following sets is a field?
a) {-1, 0, 1}
b) Polynomials
c) 2 X 2 Matrices
d) Complex Numbers

Solution begins on next page

Solution

For an algebraic structure (set of objects – numbers, matrices, etc) to be a field, it must satisfy the six field axioms:

Let x, y, and z be members of the set A

1. The set must be closed under addition and multiplication.
   $x + y$ and $x \cdot y$ are members of the set.
2. Addition and multiplication are commutative for members of the set.
   $x + y = y + x$ and $x \cdot y = y \cdot x$
3. Addition and multiplication are associative for members of the set.
   $(x + y) + z = x + (y + z)$ and $(x \cdot y) \cdot z = x \cdot (y \cdot z)$
4. There exist an additive identity element (0) and a multiplicative identity element (1) such that
   $x + 0 = x$ and $x \cdot 1 = x$
5. There exist an additive and multiplicative inverses such that
   $x + (-x) = 0$ and $x \cdot \frac{1}{x} = 1$
6. Multiplication over addition is distributive
   $x \cdot (y + z) = x \cdot y + x \cdot z$

Let's try out each of the answer choices:

The set {-1, 0, 1} is not a field since it violates the closure property (field axiom #1). $1 + 1 = 2$ which is not a member of the set.

The set "Polynomials" violates the multiplicative inverse property (field axiom #5) since a polynomial like $2x^2 + 3$ would have as its inverse $\frac{1}{2x^2 + 3}$. This is not a polynomial since polynomials are not allowed to have variables in the denominator of a fraction.

The set "2 X 2 Matrices" violates the commutative property of multiplication (#2), the associative property of multiplication (#3), and the multiplicative inverse property (#5). Reversing the order in which matrices are multiplied or grouping them differently in multiplication would yield different products. Also, any matrix that has a determinant of 0 does not have a multiplicative inverse.

The set "Complex Numbers" (**a + bi**, in which **a** is a Real Number and **i** represents the imaginary number) satisfies all the field axioms, so it is a field.

The answer is D.

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