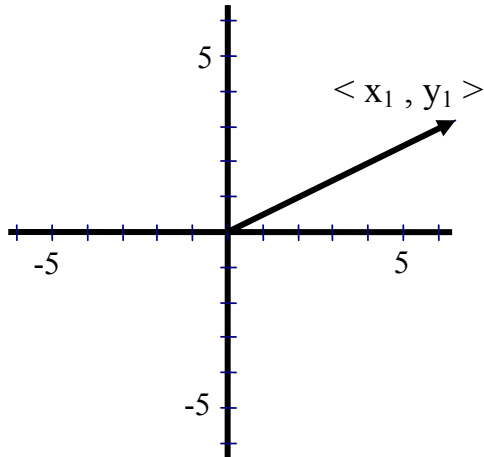


Sample Constructed Response Question from *Preparing for the CSET – Mathematics Subtest 1*

Let $\mathbf{v}_1 = \langle x_1, y_1 \rangle$ denote a vector in the xy -plane with initial point $(0, 0)$ and terminal point (x_1, y_1) as shown below.

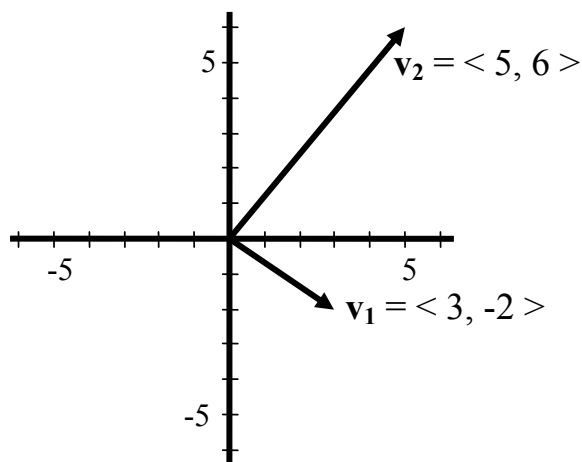


- A. Draw $\mathbf{v}_1 = \langle 3, -2 \rangle$ and $\mathbf{v}_2 = \langle 5, 6 \rangle$ and find their dot products.
- B. If \mathbf{u} lies on the line $y = x$, and \mathbf{v} lies on the line $y = -x$, show that $\mathbf{u} \bullet \mathbf{v} = 0$.
- C. If \mathbf{u} and \mathbf{v} lie on perpendicular lines, show that the dot product $\mathbf{u} \bullet \mathbf{v} = 0$.

Solution begins on next page

Solution

- A. When a vector is given in “component form”, draw the vector as an arrow with the head (terminal point) at the given point, and the tail (initial point) at the origin, as shown below.



To find the dot product $\mathbf{v}_1 \bullet \mathbf{v}_2$, multiply the x-components together and the y-components together, then add the two products.

$$\mathbf{v}_1 \bullet \mathbf{v}_2 = (3 \cdot 5) + (-2 \cdot 6) = 15 + -12 = 3$$

So the dot product $\mathbf{v}_1 \bullet \mathbf{v}_2 = 3$

- B. Since \mathbf{u} lies on the line $y = x$, its x- and y- components are equal. Its y-component is whatever its x-component is, so we can say that \mathbf{u} has the form $\langle x_1, x_1 \rangle$.

Since \mathbf{v} lies on the line $y = -x$, its x- and y- components are additive inverses. Its y-component is the negative of whatever its x-component is, so we can say that \mathbf{v} has the form $\langle x_2, -x_2 \rangle$.

$$\mathbf{u} \bullet \mathbf{v} = (x_1 \cdot x_2) + (x_1 \cdot -x_2) = x_1x_2 + -x_1x_2 = 0$$

$$\mathbf{u} \bullet \mathbf{v} = 0$$

C. Remember that if two lines are perpendicular, then their slopes are negative reciprocals.

Let vector $\mathbf{u} = \langle x_1, y_1 \rangle$. Since this names a vector with initial point $(0, 0)$ and terminal point (x_1, y_1) , then its slope is $m = \frac{\text{rise}}{\text{run}} = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$.

If the slope of \mathbf{u} is $\frac{y_1}{x_1}$, then the slope of \mathbf{v} is the negative reciprocal, $-\frac{x_1}{y_1}$.

Since the numerator of the slope represents the y-component, and the denominator represents the x-component, then $\mathbf{v} = \langle y_1, -x_1 \rangle$.

$$\text{So, } \mathbf{u} \bullet \mathbf{v} = (x_1 \cdot y_1) + (y_1 \cdot -x_1) = x_1y_1 + -y_1x_1 = 0$$

Technically, since we do not know how long \mathbf{v} is, it can be any scalar multiple of $\langle y_1, -x_1 \rangle$, so we should say that $\mathbf{v} = \langle ny_1, -nx_1 \rangle$, but this gives the same result:

$$\mathbf{u} \bullet \mathbf{v} = (x_1 \cdot ny_1) + (y_1 \cdot -nx_1) = nx_1y_1 + -ny_1x_1 = 0$$

This sample problem and solution was our gift to you just for taking the time to visit CSETMath.com. If you like what you have seen, please select the ["Order Information"](#) link, and choose the *Preparing for the CSET – Mathematics Subtest 1* workbook which contains thirty multiple choice and four constructed response questions with solutions as detailed as this one. For additional practice on the constructed response questions choose *Preparing for the CSET Mathematics — Subtest 1 Constructed Response Workbook*. Each Constructed Response Workbook contains 20 constructed response questions (with detailed solutions) which were carefully chosen to align with the Subject Matter Requirements.

Prepare To Pass With CSETMath
The smart way to study.