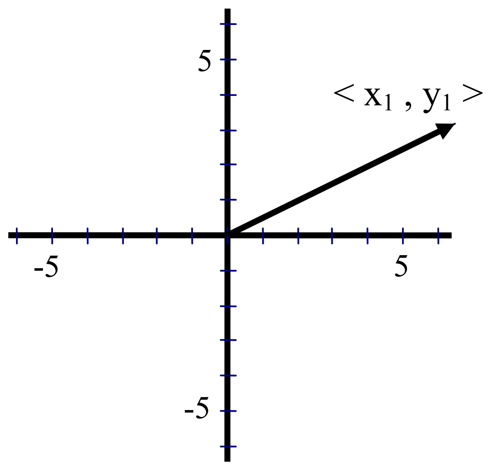
Sample Constructed Response Question from *Preparing for the CSET – Mathematics Subtest 1*

Let $\mathbf{v_1} = < x_1 , y_1 >$ denote a vector in the xy-plane with initial point (0, 0) and terminal point (x_1 , y_1) as shown below.



A. Draw $\mathbf{v_1} = < 3, -2 >$ and $\mathbf{v_2} = < 5, 6 >$ and find their dot products.

B. If $\mathbf{u}$ lies on the line $y = x$, and $\mathbf{v}$ lies on the line $y = -x$, show that $\mathbf{u} \bullet \mathbf{v} = 0$.

C. If $\mathbf{u}$ and $\mathbf{v}$ lie on perpendicular lines, show that the dot product $\mathbf{u} \bullet \mathbf{v} = 0$.

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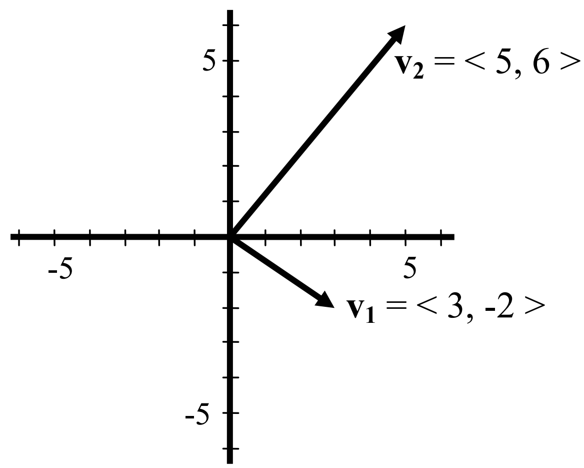
Solution

A. When a vector is given in "component form", draw the vector as an arrow with the head (terminal point) at the given point, and the tail (initial point) at the origin, as shown below.



To find the dot product $\mathbf{v_1} \bullet \mathbf{v_2}$, multiply the x-components together and the y-components together, then add the two products.

$$\mathbf{v_1} \bullet \mathbf{v_2} = (3 \cdot 5) + (-2 \cdot 6) = 15 + -12 = 3$$

So the dot product $\mathbf{v_1} \bullet \mathbf{v_2} = 3$

B. Since **u** lies on the line $y = x$, its x- and y- components are equal. Its y-component is whatever its x-component is, so we can say that **u** has the form $< x_1, x_1 >$.

Since **v** lies on the line $y = -x$, its x- and y- components are additive inverses. Its y-component is the negative of whatever its x-component is, so we can say that **v** has the form $< x_2, -x_2 >$.

$$\mathbf{u} \bullet \mathbf{v} = (x_1 \cdot x_2) + (x_1 \cdot -x_2) = x_1x_2 + -x_1x_2 = 0$$

$$\mathbf{u} \bullet \mathbf{v} = 0$$

C. Remember that if two lines are perpendicular, then their slopes are negative reciprocals.

Let vector **u** = < x_1, y_1 >. Since this names a vector with initial point (0, 0) and terminal point (x_1, y_1), then its slope is $m = \frac{rise}{run} = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$.

If the slope of **u** is $\frac{y_1}{x_1}$, then the slope of **v** is the negative reciprocal, $-\frac{x_1}{y_1}$.

Since the numerator of the slope represents the y-component, and the denominator represents the x-component, then **v** = < y_1, $-x_1$ >.

So, **u** • **v** = ($x_1 \cdot y_1$) + ($y_1 \cdot -x_1$) = $x_1y_1 + -y_1x_1 = 0$

Technically, since we do not know how long **v** is, it can be any scalar multiple of < y_1, $-x_1$ >, so we should say that **v** = < ny_1, $-nx_1$ >, but this gives the same result:

u • **v** = ($x_1 \cdot ny_1$) + ($y_1 \cdot -nx_1$) = $nx_1y_1 + -ny_1x_1 = 0$