Preparing for the CSET

Sample Book

Mathematics

by
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Preparing for the CSET
Sample Book Mathematics

We at CSETMath want to thank you for interest in Preparing *for the CSET - Mathematics*. This Sample Book is designed to give you an overview of what you will find in the full version of each book. There are three books available:

Preparing for the CSET Multiple Subject--Mathematics,
Preparing for the CSET Mathematics--Subtest I,
Preparing for the CSET Mathematics--Subtest II.

Each workbook contains a complete practice examination with detailed solutions designed to prepare you to pass the CSET Mathematics Exam. By completing these books you will familiarize yourself with the Subject Matter Requirements of the CSET.

This sample book contains multiple choice and constructed response questions and solutions that are representative of those that are available in each of the three complete practice books.

If you like what you have seen, please visit our website at www.CSETMath.com. The website contains complete descriptions of each book as well as ordering information.

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Good luck as you prepare to pass with CSETMath.

Sample Multiple Choice Questions

Preparing for the CSET – Multiple Subject Mathematics

Which of the following sets is not closed under addition?

a) Natural Numbers
b) Whole Numbers
c) Integers
d) Odd Integers

Preparing for the CSET – Mathematics Subtest l

Which of the following sets is a field?

a) {-1, 0, 1}
b) Polynomials
c) 2 X 2 Matrices
d) Complex Numbers

Preparing for the CSET – Mathematics Subtest ll

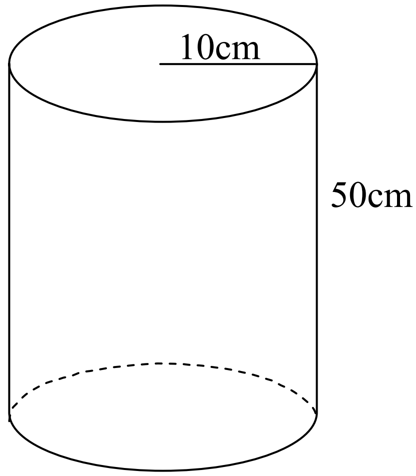
A bag contains 5 red, 4 black, and 6 blue marbles. If 4 marbles are chosen at random, what is the probability of choosing 2 red and 2 blue?

a) .0110
b) .1099
c) .1333
d) .2667

Sample Constructed Response Questions

Preparing for the CSET – Multiple Subject Mathematics

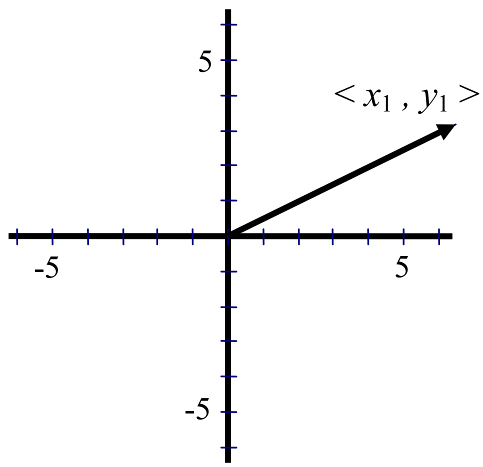
Find the surface area of a cylinder with radius 10cm and height 50cm.
Use 3.14 as π.



Sample Constructed Response Questions

Preparing for the CSET – Mathematics Subtest 1

Let $\boldsymbol{v_1} = < x_1 , y_1 >$ denote a vector in the xy-plane with initial point (0, 0) and terminal point (x_1 , y_1) as shown below.



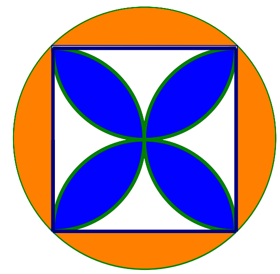
A. Draw $\boldsymbol{v_1} = < 3, -2 >$ and $\boldsymbol{v_2} = < 5, 6 >$ and find their dot products.

B. If $\boldsymbol{u}$ lies on the line $y = x$, and $\boldsymbol{v}$ lies on the line $y = -x$, show that $\boldsymbol{u} \bullet \boldsymbol{v} = 0$.

C. If $\boldsymbol{u}$ and $\boldsymbol{v}$ lie on perpendicular lines, show that the dot product $\boldsymbol{u} \bullet \boldsymbol{v} = 0$.

Sample Constructed Response Questions

Preparing for the CSET – Mathematics Subtest ll



Show that the area outside the square (Orange Area) is equal to the shaded area inside the square (Blue Area)
The Orange Area is defined as the area between a circle and an inscribed square.
The Blue Area is the overlapping area of four congruent semi-circles with each side of the square as a diameter of the semi-circle.

Solutions

Preparing for the CSET – Multiple Subject Mathematics
Which of the following sets is not closed under addition?
a) Natural Numbers
b) Whole Numbers
c) Integers
d) Odd Integers

Let's begin by talking about the sets of Real Numbers.
Natural Numbers are the counting numbers: {1, 2, 3, . . . }.
Whole Numbers are the Natural Numbers and 0: {0, 1, 2, 3, . . .}
Integers are the Whole Numbers and their negatives: {. . ., -2, -1, 0, 1, 2, . . .}

The "Closure Property of Real Numbers" is the one property that is most often skipped in beginning algebra classes. A set of numbers is said to be closed under an operation if when you begin the operation with numbers from that set, your answer is also a member of that set.

For example, the set "Natural Numbers" is closed under multiplication since whenever you multiply two Natural Numbers, the answer is also a Natural Number. The set "Natural Numbers" is not closed under division since it is possible to divide two Natural Numbers and get an answer that is not a Natural Number (if we divided 4 by 5 we would get 4/5 which is not a Natural Number). We need only one exception to prove that a statement is false.

In this problem, we are adding two numbers together. Let's try out each of the answer choices:

a) Any time we add two Natural Numbers together, we get a Natural Number. Therefore the set "Natural Numbers" **is** closed under addition.
b) Any time we add two Whole Numbers together, we get a Whole Number. Therefore the set "Whole Numbers" **is** closed under addition.
c) Any time we add two Integers together, we get an Integer. Therefore the set "Integers" **is** closed under addition.
d) Any time we add two Odd Integers together, the answer is not an Odd Integer – it is an even integer: (3 + 5 = 8). So the set "Odd Integers" **is not** closed with respect to addition since whenever you add two Odd Integers together, the result is not always an Odd Integer. (In fact the answer never is odd, but we needed to show only one counter example to disprove closure.)

Solution

Preparing for the CSET – Mathematics Subtest I

Which of the following sets is a field?

a) {-1, 0, 1}
b) Polynomials
c) 2 X 2 Matrices
d) Complex Numbers

For an algebraic structure (set of objects – numbers, matrices, polynomials, etc.) to be a field, it must satisfy the six field axioms:

Let *x, y,* and *z* be members of the set *A*

1. The set must be closed under addition and multiplication.
   $x + y$ and $x \cdot y$ are members of the set.
2. Addition and multiplication are commutative for members of the set.
   $x + y = y + x$ and $x{\cdot}y = y{\cdot}x$
3. Addition and multiplication are associative for members of the set.
   $(x + y) + z = x + (y + z)$ and $(x{\cdot}y){\cdot}z = x{\cdot}(y{\cdot}z)$
4. There exist an additive identity element (0) and a multiplicative identity element (1) such that
   $x + 0 = x$ and $x \cdot 1 = x$
5. There exist an additive and multiplicative inverses such that
   $x + (-x) = 0$ and $x \cdot \frac{1}{x} = 1$
6. Multiplication over addition is distributive
   $x \cdot (y + z) = x \cdot y + x \cdot z$

Let's try out each of the answer choices:

The set {-1, 0, 1} is not a field since it violates the closure property (field axiom #1). $1 + 1 = 2$, which is not a member of the set.

The set "Polynomials" violates the multiplicative inverse property (field axiom #5) since a polynomial like $2x^2 + 3$ would have as its multiplicative inverse $\frac{1}{2x^2+3}$. This is not a polynomial since polynomials are not allowed to have variables in the denominator of a fraction.

The set "2 X 2 Matrices" violates the commutative property of multiplication (#2), the associative property of multiplication (#3), and the multiplicative inverse property (#5). Reversing the order in which matrices are multiplied or grouping them differently in multiplication would yield different products. Also, any matrix that is not square or has a determinant of 0 does not have a multiplicative inverse.

The set “Complex Numbers” ($a + bi$, in which a is a Real Number and i represents the imaginary number) satisfies all the field axioms, so it is a field.

The answer is D.

Solution

Preparing for the CSET – Mathematics Subtest ll

A bag contains 5 red, 4 black, and 6 blue marbles. If 4 marbles are chosen at random, what is the probability of choosing 2 red and 2 blue?

a) .0110
b) .1099
c) .1333
d) .2667

Probability is defined as a fraction in which the numerator is the number of ways that the successful outcome can occur and the denominator is the total number of outcomes.

First find the numerator:

In this problem, success is defined as choosing 2 red and 2 blue marbles. The 2 red marbles are chosen from the 5 red marbles that are available, and the 2 blue marbles are chosen from the 6 blue marbles that are available.

We use Combinations to solve this problem:

${}_nC_r = \dfrac{n!}{r!(n-r)!}$ where **n** is the total items available, **r** is how many we want, and the **!** means "factorial": for example, 6! = 6·5·4·3·2·1 = 720

For our numerator, we have

$${}_5C_2 \cdot {}_6C_2 = \frac{5!}{2!(5-2)!}\cdot\frac{6!}{2!(6-2)!} = \frac{5!}{2!(3)!}\cdot\frac{6!}{2!(4)!} = 10\cdot 15 = 150$$

For the denominator, we also use combinations; but we use the total number of items without regard to what type they are. In other words, we are choosing 4 total marbles from 15 total marbles available.

$${}_{15}C_4 = \frac{15!}{4!(15-4)!} = \frac{15!}{4!(11)!} = 1365$$

So the probability of choosing 2 red and 2 blue marbles is

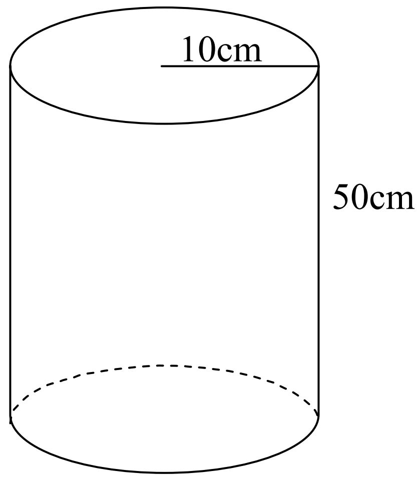
$$\frac{{}_5C_2 \cdot {}_6C_2}{{}_{15}C_4} = \frac{150}{1365} = .1099$$ which is answer B.

These values could be calculated by multiplying out the factorials, but it is much faster using the ${}_nC_r$ button on the calculator. On a TI-83, enter the value of **n**, press the "MATH" button, arrow over to "PRB" on the display, then arrow down to "${}_nC_r$" on the display. Press "ENTER", then enter the value of **r** and press "ENTER".

Solution

Preparing for the CSET – Multiple Subject Mathematics

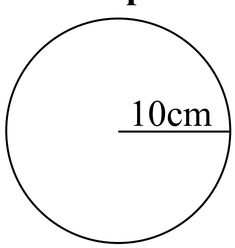
Find the surface area of a cylinder with radius 10cm and height 50cm.
Use 3.14 as π.



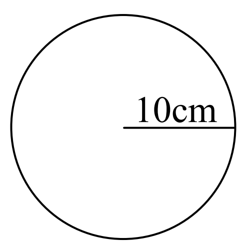
The surface area of a solid object is the area of its surfaces. That bit of circular reasoning doesn't help much, so let's think of the cylinder as a soup can and list all of its surfaces.

Take a can opener and remove the top and bottom from the can. We have 2 circles, each with a radius of 10.

Top



Bottom



The formula for the Area of a circle is $A = \pi \cdot r^2$

So the Area of **each** of these circles is $A = 3.14 \cdot (10\text{cm})^2 = 3.14 \cdot 100 = 314\ \text{cm}^2$

Since there is a top and bottom to our can, the area of both circles is

Area (both circles) $= 2 \cdot 314\text{cm}^2 = 628\text{cm}^2$

Now for the side of the can, think about cutting off the label and unrolling it. We get a rectangle where the height is the height of the can and the base is the circumference of the circle (the distance around the lid).

The formula for the Circumference of a circle is $C = 2 \cdot \pi \cdot r$

For our lid it is $C = 2 \cdot 3.14 \cdot 10\text{cm} = 62.8$ cm

So now our rectangle can be labeled:

62.8cm

50cm

The formula for the Area of a rectangle is $A = b \cdot h$ For the label of our soup can, that is $A = 62.8\text{cm} \cdot 50\text{cm} = 3140\ \text{cm}^2$

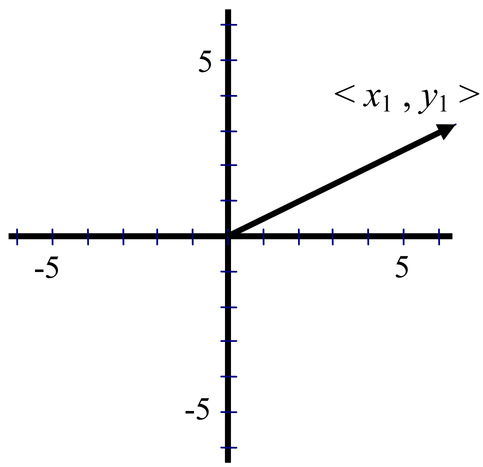
So in total, the surface area is the area of the top and bottom circles plus the area of the rectangular label.

$SA = 628\ \text{cm}^2 + 3140\ \text{cm}^2 = 3768\text{cm}^2$

Solution

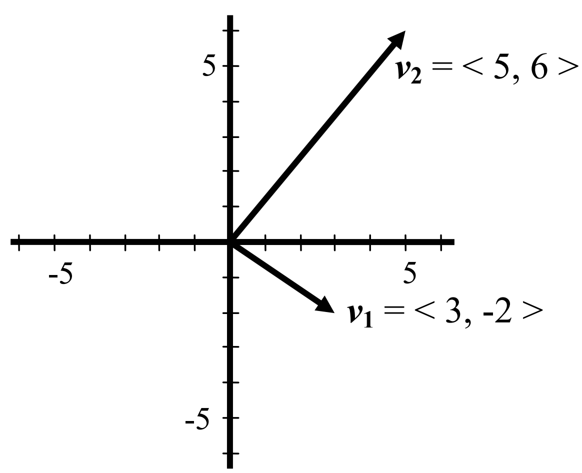
Preparing for the CSET – Mathematics Subtest 1

Let $\boldsymbol{v_1} = < x_1 , y_1 >$ denote a vector in the xy-plane with initial point (0, 0) and terminal point (x_1 , y_1) as shown below.



A. Draw $\boldsymbol{v_1} = < 3, -2 >$ and $\boldsymbol{v_2} = < 5, 6 >$ and find their dot products.

When a vector is given in "component form", draw the vector as an arrow with the head (terminal point) at the given point, and the tail (initial point) at the origin, as shown below.



To find the dot product $\boldsymbol{v_1} \bullet \boldsymbol{v_2}$, multiply the x-components together and the y-components together, then add the two products.

$$\boldsymbol{v_1} \bullet \boldsymbol{v_2} = (3 \cdot 5) + (-2 \cdot 6) = 15 + -12 = 3$$

So the dot product $\boldsymbol{v_1} \bullet \boldsymbol{v_2} = 3$

B. If $\boldsymbol{u}$ lies on the line $y = x$, and $\boldsymbol{v}$ lies on the line $y = -x$, show that $\boldsymbol{u} \bullet \boldsymbol{v} = 0$.

Since **I** lies on the line $y = x$, its x- and y- components are equal. Its y-component is whatever its x-component is, so we can say that $\boldsymbol{u}$ has the form $< x_1, x_1 >$.
Since $\boldsymbol{v}$ lies on the line $y = -x$, its x- and y- components are additive inverses. Its y-component is the negative of whatever its x-component is, so we can say that $\boldsymbol{v}$ has the form $< x_2, -x_2 >$.

$$\boldsymbol{u} \bullet \boldsymbol{v} = (x_1 \cdot x_2) + (x_1 \cdot -x_2) = x_1x_2 + -x_1x_2 = 0$$
$$\boldsymbol{u} \bullet \boldsymbol{v} = 0$$

C. If $\boldsymbol{u}$ and $\boldsymbol{v}$ lie on perpendicular lines, show that the dot product $\boldsymbol{u} \bullet \boldsymbol{v} = 0$.

Remember that if two lines are perpendicular, then their slopes are negative reciprocals.
Let vector $\boldsymbol{u} = < x_1, y_1 >$. Since this names a vector with initial point (0, 0) and terminal point (x_1, y_1), then its slope is $m = \frac{rise}{run} = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$.

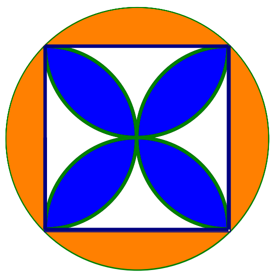
If the slope of $\boldsymbol{u}$ is $\frac{y_1}{x_1}$, then the slope of $\boldsymbol{v}$ is the negative reciprocal, $-\frac{x_1}{y_1}$.
Since the numerator of the slope represents the y-component, and the denominator represents the x-component, then $\boldsymbol{v} = < y_1, -x_1 >$.
So, $\boldsymbol{u} \bullet \boldsymbol{v} = (x_1 \cdot y_1) + (y_1 \cdot -x_1) = x_1y_1 + -y_1x_1 = 0$

Technically, since we do not know how long $\boldsymbol{v}$ is, it can be any scalar multiple of $< y_1, -x_1 >$, so we should say that $\boldsymbol{v} = < ny_1, -nx_1 >$, but this gives the same result:
$\boldsymbol{u} \bullet \boldsymbol{v} = (x_1 \cdot ny_1) + (y_1 \cdot -nx_1) = nx_1y_1 + -ny_1x_1 = 0$

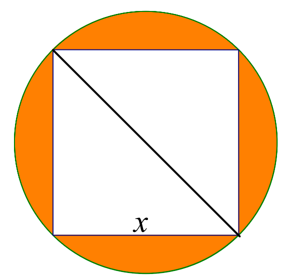
Solution

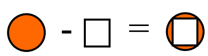
Show that the area of the region shaded orange outside the square is equal to the total area of the regions shaded blue inside the square.

The Orange Area is defined as the area between a circle and the inscribed square. The Blue Area is the overlapping area of four congruent semi-circles with each side of the square as a diameter of the semi-circle.



Start by finding the Orange Area.
To find the Orange Area, we must find the area of the circle and subtract from it the area of the square.





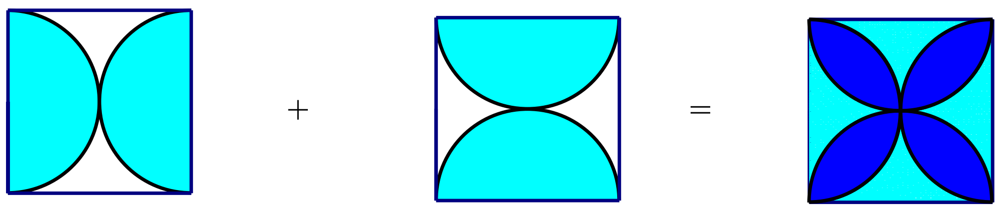
If we define x as the length of the side of the square, then the area of the square would be x^2.

The diagonal of the square is also the diameter of the circle. Using the Pythagorean Theorem, we can find its length to be $x\sqrt{2}$. The radius is ½ of the diameter, or $\frac{x\sqrt{2}}{2}$. The area of a circle is πr^2, or in this case $\pi\left(\frac{x\sqrt{2}}{2}\right)^2$, which simplifies to $\frac{\pi x^2}{2}$.

Since the Orange Area is the difference between the areas of the circle and square, it is $\frac{\pi x^2}{2} - x^2$ square units.

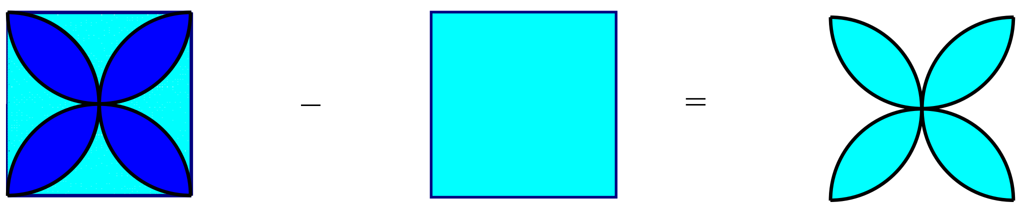
Now let's find the Blue Area.

Since the Blue area is formed by four overlapping semi-circles we should start by adding together their areas. (In these diagrams, dark shading represents areas that have been counted twice and light shading represents areas that have been counted once.)



The problem with this is that we have double counted the area that we want (the dark blue flower) and single-counted the area that we didn't even want to count once (the light blue wedges).

If we take our above result and subtract the area of a square, we lose the single-counted wedges and reduce the double counted flower to the single-count that we want.



Numerically, the radius of each semi-circle is $\frac{x}{2}$. (The side of the square is the diameter of the semi-circle, so cut it in half to get the radius.) Therefore, the total area of the 4 semicircles would be $4\left(\frac{1}{2}\left[\pi\left(\frac{x}{2}\right)^2\right]\right)$ which simplifies to $\frac{\pi x^2}{2}$. (The ½ is in there because we are looking at semi-circles.)

When we then subtract the area of the square we get $\frac{\pi x^2}{2} - x^2$ square units, which is same as the Orange Area found above.

These sample problems and solutions are our gift to you just for taking the time to visit the CSETMath.com website. If you like what you have seen, please select the "Order Now" link. Each of the *Preparing for the CSET – Mathematics* workbooks contain thirty multiple choice and four constructed response questions with solutions as detailed as this one.